Ocean Engineering 165 (2018) 69–76

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

# Abundant wave solutions of the Boussinesq equation and the (2+1)-dimensional extended shallow water wave equation

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ARTICLE INFO

*Keywords:*
Closed form wave solutions
Boussinesq equation
(2+1)-dimensional extended shallow water wave equation
New generalized *(G′/G)*-expansion method

ABSTRACT

In this article, we establish the exact wave solutions of the Boussinesq equation and the (2 + 1)-dimensional extended shallow water wave equation by applying the new generalized $(G'/G)$ -expansion method. When the condition of the fluid is such that the horizontal length scale is much greater than the vertical length scale, the shallow water equations are mostly suitable. In Ocean engineering, Boussinesq-type equations are commonly used in computer simulations for the model of water waves in shallow seas and harbors. We explained the new generalized $(G'/G)$ -expansion method to seek further general traveling wave solutions of the above mentioned equations. The traveling wave solutions attained by this method are exposed in terms of hyperbolic, trigonometric and rational functions. The shape of the obtained solutions are bell shaped soliton, kink soliton, singular kink soliton, singular soliton, singular periodic solution and compaction. This method is very influential mathematical tool for extracting exact solutions of NLEEs which frequently arise in mathematical physics, engineering sciences and many scientific real world application fields.

## 1. Introduction

The complex physical phenomena of different branch of sciences are described by the nonlinear evolution equations (NLEEs). Therefore, the investigation of the traveling wave solutions of NLEEs plays a vital role in the field of propagation of shallow water waves, meteorology, fluid mechanics, applied mathematics, theoretical physics and engineering, such as, plasma physics, solid state physics, ecology, quantum mechanics etc. Some specific nonlinear NLEEs that are useful in various fields of science and engineering are namely, the Maxwell's equation in electromagnetism, the heat equation in thermodynamics, the Schrodinger equation in quantum mechanics, the Lotka-Voltera equation in biological population dynamics, the Navier-Stokes equation in fluid dynamics etc. Exacting analytical solutions of nonlinear wave equations by applying the appropriate methods have shown a massive dynamism. It is mentionable that, there have some significant improvements in the discussion of exact solution in the recent years. Searching the exact traveling wave solutions by implementing adequate techniques and useful methods has also shown enormous essentiality. In recent years, various powerful methods for obtaining exact traveling solutions of NLEEs have been developed, such as, the Backlund transformation method (Mimura, 1978; Mimura, 1978; Liu and He, 2017), the Hirota's bilinear transformation method (Liu and He, 2017, 2018; Hirota, 1971; Liu et al., 2017a, 2017b, 2017c, 2018a, 2018b; Huang et al., 2017; Matveev and Salle, 1991; Cole, 1951; Hopf, 1950; Wazwaz, 2007, 2017; Malfliet, 1992; Kumar and Dayal, 2015; Kabir, 2017; Khan and Akbar, 2013; Hosseini et al., 2017; Irendaoreji, 2004; Zayed and Al-Nowehy, 2017; Sonmezoglu et al., 2017; Malwe et al., 2015; Nofal, 2016; Jesmin Akter and Akbar, 2015; Ali et al., 2017; Roshid, 2017; Hossain et al., 2017; Hossain and Akbar, 2017; Hafez et al., 2015; Liu, 2018a, 2018b; Wang et al., 2008; Zayed and Gepreel, 2009; Feng et al., 2011; Naher et al., 2011; Abazari and Abazari, 2011; Naher and Abdullah, 2012, 2013; AkbarAli, 2013; Zhang et al., 2010; Akbar et al., 2012), the Darboux transformation method (Huang et al., 2017; Matveev and Salle, 1991), the Cole-Hopf transformation method (Liu et al., 2017b, 2017c; Cole, 1951; Hopf, 1950), the tanh method (Liu and He, 2018; Wazwaz, 2017), the tanh-coth method (Matveev and Salle, 1991; Cole, 1951; Hopf, 1950; Malfliet, 1992; Wazwaz, 2007; Kumar and Dayal, 2015), the exp-function method (Wazwaz, 2007, 2017; Malfliet, 1992; Kabir, 2017; Khan and Akbar, 2013; Hosseini et al., 2017), the *F*-expansion method (Kumar and Dayal, 2015; Irendaoreji, 2004), the Jacobi elliptic function method (Kabir, 2017; Zayed and Al-Nowehy, 2017) the extended Jacobi elliptic function method (Khan and Akbar, 2013; Sonmezoglu et al., 2017), the Riccati equation method (Hosseini et al., 2017; Malwe et al., 2015), the simple equation method (Irendaoreji, 2004; Nofal, 2016), the modified simple equation method

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https://doi.org/10.1016/j.oceaneng.2018.07.025
Received 18 March 2018; Received in revised form 17 May 2018; Accepted 8 July 2018
Available online 17 July 2018
0029-8018/ 

(Zayed and Al-Nowehy, 2017; Sonmezoglu et al., 2017; Malwe et al., 2015; Nofal, 2016; Jesmin Akter and Akbar, 2015; Ali et al., 2017; Roshid, 2017; Hossain et al., 2017; Hossain and Akbar, 2017), the exp (-ϕ(ξ))-method (Ali et al., 2017; Hafez et al., 2015), the multiple exp-function method (Liu et al., 2018a), ansatz functions (Liu, 2018a) and others (Liu et al., 2018b; Liu, 2018b) etc. To analyze exact traveling wave solutions of NLEEs, another important method namely, the (G'/G) -expansion method was introduced by Wang et al. (Roshid, 2017; Liu, 2018b). Consentaneous Wang et al. many researchers applied this method to investigate different type of NLEEs to form the exact solutions (Hossain et al., 2017; Hossain and Akbar, 2017; Hafez et al., 2015; Liu et al., 2018a, 2018b; Liu, 2018a; Zayed and Gepreel, 2009; Feng et al., 2011; Naher et al., 2011; Abazari and Abazari, 2011; Naher and Abdullah, 2012; AkbarAli, 2013). After that, several improvement of the (G'/G) -expansion method have been mentioned and advanced by diverse groups of researcher. Different extent and elevation shows the effectiveness and genuineness of this method. Zhang et al. (Liu, 2018b; Zhang et al., 2010) extended the (G'/G) -expansion method namely the improved (G'/G) -expansion method. Recently, the generalized and improved (G'/G) -expansion method with additional parameter established by Akbar et al. (Wang et al., 2008; Akbar et al., 2012). Very recently, the new generalized (G'/G) -expansion method has been introduced (Zayed and Gepreel, 2009; Naher and Abdullah, 2013) which is more explainable and easy for a class of NLEEs to attain different type of new traveling wave solutions with additional parameters. The objective of this article is construct exact solutions for nonlinear evolution equations in mathematical physics through the Boussinesq equation and the $(2+1)$ -dimensional extended shallow water wave equation by means of the new generalized $(G'/G)$ -expansion method.

## 2. Explanation of the new generalized $(G'/G)$-expansion method

In this section, we present the new approach of the generalized $(G'/G)$-expansion method (Feng et al., 2011; Naher et al., 2011; Abazari and Abazari, 2011; Naher, 2015; Alam et al., 2014; Alam and Akbar, 2015) for investigating traveling wave solution to nonlinear evolution equation.

Consider the general nonlinear evolution equation in the following form

$$F(u,\ u_t,\ u_x,\ u_{tt},\ u_{tx},\ u_{xx},\ \dots)=0 \tag{1}$$

where $u=u(x,t)$ is an unidentified function, $F$ is a polynomial in $u(x,t)$ which has highest order derivatives and nonlinear terms and the order of this method is as follows:

Step 1 We consider the combination of spatial variable $x$ and temporal variable $t$ be the variable $\eta$ as follows:

$$u(x,\ t)=u(\eta),\ \eta=x\pm vt \tag{2}$$

where $v$ is the speed of the traveling wave. Through equation (2), equation (1) will be changed into an ordinary differential equation (ODE) as follows:

$$R(u,\ u',\ u'',\ u''',\ \dots)=0 \tag{3}$$

where $R$ is a polynomial of $u$ and its derivatives and the superscripts indicate the ordinary derivatives with respect to $\eta$.

Step 2 Conforming to possibility, equation (3) can be integrated term by term and integral constant maybe zero for simplicity.

Step 3 We consider the traveling wave solution of equation (3) as follows:

$$u(\eta)=\sum_{k=0}^{M}a_k(d+H)^k+\sum_{k=1}^{M}b_k(d+H)^{-k} \tag{4}$$

where either $a_M$ or $b_M$ may be zero but both $a_M$ and $b_M$ could not be zero at a time. $a_k\ (k=0,\ 1,\ 2,\ 3,\ \dots M)$, $b_k\ (k=1,\ 2,\ 3,\ \dots M)$ and $d$ are arbitrary constant to be determined, such that

$$H=(G'/G) \tag{5}$$

where $G=G(\eta)$ satisfies the following auxiliary nonlinear ODE

$$AGG''-BGG'-EG^2-C(G')^2=0 \tag{6}$$

prime indicates the derivative with respect to $\eta$ and $A$, $B$ $C$, $E$ are parameters.

Step 4 The positive integer $M$ is determined by taking the homogeneous balance between the highest order nonlinear term and the highest order derivative appearing in equation (3).

Step 5 Inserting equations (4) and (6) along with equation (5) into equation (3) with the value of $M$ obtained in step 4, we obtain polynomial in $(d+H)^M$, $(M=0,\ 1,\ 2,\ \dots)$ and $(d+H)^{-M}$, $(M=1,\ 2,\ 3,\ \dots)$. Then we collect each coefficient of the resulted polynomial is equal to zero and obtain a class of algebraic equations for $a_k(k=0,\ 1,\ 2,\ \dots,\ M)$, $b_k(k=1,\ 2,\ 3,\dots,\ M)$, $d$ and $v$.

Step 6 The general solution of equation (5) is well familiar to us, replacing the value of $a_k(k=0,\ 1,\ 2,\ \dots,\ M)$, $b_k(k=1,\ 2,\ \dots,\ M)$, $d$ and $v$ into equation (4), then we find more general type and new exact traveling wave solution of the nonlinear evolution equation (1).

Step 7 With the help of general solution of equation (6) we attain the following solution of equation (5).

Family 1: When $B\neq 0$, $\varphi=A-C$ and $Q=B^2+4E(A-C)>0$,

$$H(\eta)=\frac{B}{2\varphi}+\frac{\sqrt{Q}}{2\varphi}\frac{l\sinh\left(\frac{\sqrt{Q}}{2A}\cdot\eta\right)+m\cosh\left(\frac{\sqrt{Q}}{2A}\cdot\eta\right)}{l\cosh\left(\frac{\sqrt{Q}}{2A}\cdot\eta\right)+m\sinh\left(\frac{\sqrt{Q}}{2A}\cdot\eta\right)} \tag{7}$$

Family 2: When $B\neq 0$, $\varphi=A-C$ and $Q=B^2+4E\,(A-C)<0$,

$$H(\eta)=\frac{B}{2\varphi}+\frac{\sqrt{-Q}}{2\varphi}\frac{-l\sin\left(\frac{\sqrt{-Q}}{2A}\cdot\eta\right)+m\cos\left(\frac{\sqrt{-Q}}{2A}\cdot\eta\right)}{l\cos\left(\frac{\sqrt{-Q}}{2A}\cdot\eta\right)+m\sin\left(\frac{\sqrt{-Q}}{2A}\cdot\eta\right)}. \tag{8}$$

Family 3: When $B\neq 0$, $\varphi=A-C$ and $Q=B^2+4E(A-C)=0$,

$$H(\eta)=\frac{B}{2\varphi}+\frac{m}{l+m\,\eta} \tag{9}$$

Family 4: When $B=0$, $\varphi=A-C$ and $P=\ \varphi E>0$

$$H(\eta)=\frac{\sqrt{P}}{\varphi}\frac{l\sinh\left(\frac{\sqrt{P}}{A}\cdot\eta\right)+m\cosh\left(\frac{\sqrt{P}}{A}\cdot\eta\right)}{l\cosh\left(\frac{\sqrt{P}}{A}\cdot\eta\right)+m\sinh\left(\frac{\sqrt{P}}{A}\cdot\eta\right)} \tag{10}$$

Family 5: When $B=0$, $\varphi=A-C$ and $P=\ \varphi E<0$,

$$H(\eta)=\frac{\sqrt{-P}}{\varphi}\frac{-l\sin\left(\frac{\sqrt{-P}}{A}\cdot\eta\right)+m\cos\left(\frac{\sqrt{-P}}{A}\cdot\eta\right)}{l\cos\left(\frac{\sqrt{-P}}{A}\cdot\eta\right)+m\sin\left(\frac{\sqrt{-P}}{A}\cdot\eta\right)}. \tag{11}$$

The determination of the solution is accomplished.

## 3. Formation of the solutions

In this section, we have established some further general and new closed form wave solutions to the Boussinesq equation and the (2 + 1)-dimensional extended shallow water wave equation by using the new generalized $(G'/G)$ -expansion method.

### 3.1. The Boussinesq equation

The Boussinesq equation can be represented as:

$$u_{tt} - u_{xx} - (u^2)_{xx} - \beta u_{xxxx} = 0 \tag{12}$$

In 1872, the Boussinesq equation was established by Joseph Boussinesq to describe the propagation of small amplitude, long waves on the surface of shallow water. The Boussinesq estimation for water wave considers the vertical structure of the horizontal and vertical flow velocity. This results in partial differential equations, called Boussinesq-type equations, which incorporate frequency dispersion. In coastal engineering, Boussinesq-type equations are frequently used in computer models for the simulation of water waves in shallow seas and harbors.

Equation (12) reduced to the following ODE by using the transformation $u(x, t) = u(\eta)$, where $\eta = x - vt$,

$$(v^2 - 1)u' - (u^2)' - \beta u''' = 0 \tag{13}$$

Integrating equation (13) with respect to $\eta$ twice, we obtain

$$(v^2 - 1)u - u^2 - \beta u'' + c = 0 \tag{14}$$

where $c$ is an integrating constant which to be determined. To obtain the value of $M$, applying the homogeneous balance between $u^2$ and $u''$ in equation (14), we obtain $M = 2$. Consequently, we obtain the solution of equation (14) in the following form

$$u(\eta) = a_0 + a_1(d + H) + a_2(d + H)^2 + b_1(d + H)^{-1} + b_2(d + H)^{-2} \tag{15}$$

where $a_0,\ a_1,\ a_2,\ b_1,\ b_2$ and $d$ are constants. Substituting equation ns (5) and (15) into equation (14), we observed that the left hand side of equation (14) is converted into the polynomial of $(G'/G)^M$, $(M = 0,\ 1,\ 2,\ 3,\ \ldots)$ and $(G'/G)^{-M}$, $(M = 1,\ 2,\ 3,\ \ldots)$. We collect each coefficient of these resulted polynomials is equal to zero and attain a group of algebraic equations and solve the algebraic equations by the algebraic computation software, like Maple, for the constants $a_0,\ a_1,\ a_2,\ b_1,\ b_2,\ v,\ c,\ d$ and found three sets of solutions as follows:

Set 1:

$$\begin{aligned} a_0 &= \frac{(v^2 - 1)A^2 + 8\beta E\varphi + 2\beta B^2}{2A^2},\ a_1 = 0,\ a_2 = -\frac{6\beta\varphi^2}{A^2},\ b_1 = 0,\ b_2 \\ &= -\frac{3\beta(16E^2\varphi^2 + 8B^2E\varphi + B^4)}{8A^2\varphi^2},\ d = -\frac{B}{2\varphi},\ v = v,\ c \\ &= -\frac{(v^2 - 1)^2A^4 - 256\beta^2E^2\varphi^2 - 128\beta^2B^2E\varphi - 16\beta^2B^4}{4A^4}, \end{aligned} \tag{16}$$

Set 2:

$$\begin{aligned} a_0 &= -\frac{(1 - v^2)A^2 + 12\beta d^2\varphi^2 + 12\beta dB\varphi - 8\beta P + \beta B^2}{2A^2}, \\ a_1 &= \frac{6\beta(2d\varphi^2 + B\varphi)}{A^2}, \\ a_2 &= -\frac{6\beta\varphi^2}{A^2}, \\ b_1 &= 0, \\ b_2 &= 0, \\ d &= d, \\ v &= v, \\ c &= -\frac{(v^2 - 1)^2A^4 - 16\beta^2P^2 - 8\beta^2B^2P - \beta^2B^4}{4A^4} \end{aligned} \tag{17}$$

Set 3:

$$\begin{aligned} a_0 &= -\frac{(1 - v^2)A^2 + 12\beta d^2\varphi^2 + 12\beta dB\varphi - 8\beta P + \beta B^2}{2A^2}, \\ a_1 &= 0,\ a_2 = 0,\ v = v,\ d = d,\ b_1 = \frac{6\beta(2d^3\varphi^2 + 3d^2B\varphi - 1dP + B^2d - BE)}{A^2}, \\ b_2 &= \frac{-6\beta\{d^4\varphi^2 + 2d^3B\varphi - 2d^2P + (Bd - E)^2\}}{A^2}, \\ c &= -\frac{(v^2 - 1)^2A^4 - 16\beta^2P^2 - 8\beta^2B^2P - \beta^2B^4}{4A^4} \end{aligned} \tag{18}$$

Where $\varphi = A - C,\ P = \varphi E$, and $A,\ B,\ C,\ E$ are parameters.

For set 1: When $B \neq 0,\ \varphi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, substituting the values of the constants arranged in equation (16) into equation (15), as well as equation (7) and simplifying, we obtain the following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ respectively

$$u_{1_1} = r_1 - \frac{6\beta Q}{4A^2}\coth^2\left(\frac{\sqrt{Q}}{2\varphi}\eta\right) - r_2\tanh^2\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)$$

$$u_{1_2} = r_1 - \frac{6\beta Q}{4A^2}\tanh^2\left(\frac{\sqrt{Q}}{2\varphi}\eta\right) - r_2\coth^2\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)$$

In the similarly fashion, replacing the values of the constants arranged in equation (16) into equation (15), as well as equations (8)–(11) and facilitating, we attain respectively following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ as follows:

$$u_{1_3} = r_1 + \frac{6\beta Q}{4A^2}\cot^2\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right) + r_2\tan^2\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right),$$

$$u_{1_4} = r_1 + \frac{6\beta Q}{4A^2}\tan^2\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right) + r_2\cot^2\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right),$$

$$u_{1_5} = r_1 - \frac{6\beta\varphi^2}{A^2\eta^2}\left(\frac{1}{\eta}\right)^2 - r_3\left(\frac{1}{\eta}\right)^{-2},$$

$$u_{1_6} = r_1 - \frac{6\beta\varphi^2}{A^2}\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^2 - r_3\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-2},$$

$$u_{1_7} = r_1 - \frac{6\beta\varphi^2}{A^2}\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^2 - r_3\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-2}$$

$$\begin{aligned} u_{1_8} = r_1 &- \frac{6\beta\varphi^2}{A^2}\left\{-\frac{B}{2\varphi} + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^2 \\ &- r_3\left\{-\frac{B}{2\varphi} + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-2}, \end{aligned}$$

$$\begin{aligned} u_{1_9} = r_1 &- \frac{6\beta\varphi^2}{A^2}\left\{-\frac{B}{2\varphi} - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^2 \\ &- r_3\left\{-\frac{B}{2\varphi} - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-2} \end{aligned}$$

Where $r_1 = \frac{(v^2 - 1)A^2 + 8\beta E\varphi + 2\beta B^2}{2A^2}$, $r_3 = \frac{3\beta(16E^2\varphi^2 + 8B^2E\varphi + B^4)}{8A^2\varphi^2}$.

For set 2: When $B \neq 0,\ \varphi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, replacing the values of the constants arranged in equation (17) into equation (15), along with equation (7) and simplifying, we get following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ respectively

$$\begin{aligned} u_{2_1} = s_1 &+ s_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\} \\ &- \frac{6\beta\varphi^2}{A^2}\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^2, \end{aligned}$$

$$\begin{aligned} u_{2_2} = s_1 &+ s_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\} \\ &- \frac{6\beta\varphi^2}{A^2}\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^2 \end{aligned}$$

In the same way, substituting the values of the constants arranged in equation (17) into equation (15), as well as equations (8)–(11) and simplifying, we attain respectively following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ as follows:

$$u_{23} = s_1 + s_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{-Q}}{2\varphi}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\} - \frac{6\beta\varphi^2}{A^2}\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{-Q}}{2\varphi}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^2,$$

$$u_{24} = s_1 + s_2\left\{d + \frac{B}{2\varphi} - \frac{\sqrt{-Q}}{2\varphi}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\} - \frac{6\beta\varphi^2}{A^2}\left\{d + \frac{B}{2\varphi} - \frac{\sqrt{-Q}}{2\varphi}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^2,$$

$$u_{25} = s_1 + s_2\left(d + \frac{B}{2\varphi} + \frac{1}{\eta}\right) - \frac{6\beta\varphi^2}{A^2}\left(d + \frac{B}{2\varphi} + \frac{1}{\eta}\right)^2,$$

$$u_{26} = s_1 + s_2\left\{d + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\} - \frac{6\beta\varphi}{A^2}\left\{d + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^2,$$

$$u_{27} = s_1 + s_2\left\{d + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\} - \frac{6\beta\varphi}{A^2}\left\{d + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^2,$$

$$u_{28} = s_1 + s_2\left\{d + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\} - \frac{6\beta\varphi}{A^2}\left\{d + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^2,$$

$$u_{29} = s_1 + s_2\left\{d - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\} - \frac{6\beta\varphi}{A^2}\left\{d + \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^2$$

where $s_1 = -\frac{(1-v^2)A^2 + 12\beta d^2\varphi^2 + 12\beta d\, B\varphi - 8\beta P + \beta B^2}{2A^4}$, $s_2 = \frac{6\beta(2d\varphi^2 + B\beta)}{A^2}$.

For set 3: When $B \neq 0$, $\varphi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, replacing the values of the constants arranged in equation (18) into equation (15), along with equation (7) and simplifying, we get following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ respectively

$$u_{31} = e_1 + e_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-2},$$

$$u_{32} = e_1 + e_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-2}$$

In the similar manner, substituting the values of the constants arranged in equation (18) into equation (15), as well as equations (8)–(11) and simplifying, we attain respectively following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ as follows:

$$u_{33} = e_1 + e_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{-Q}}{2\varphi}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{-Q}}{2\varphi}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-2},$$

$$u_{34} = e_1 + e_2\left\{d + \frac{B}{2\varphi} - \frac{\sqrt{-Q}}{2\varphi}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{B}{2\varphi} - \frac{\sqrt{-Q}}{2\varphi}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-2},$$

$$u_{35} = e_1 + e_2\left(d + \frac{B}{2\varphi} + \frac{1}{\eta}\right)^{-1} + e_3\left(d + \frac{B}{2\varphi} + \frac{1}{\eta}\right)^{-2},$$

$$u_{36} = e_1 + e_2\left\{d + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-2},$$

$$u_{37} = e_1 + e_2\left\{d + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-2},$$

$$u_{38} = e_1 + e_2\left\{d + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-1} + e_3\left\{d + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-2},$$

$$u_{39} = e_1 + e_2\left\{d - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\} + e_3\left\{d - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^2$$

where $e_1 = -\frac{(1-v^2)A^2 + 12\beta d^2\varphi^2 + 12\beta dB\varphi - 8\beta P + \beta B^2}{2A^2}$, $e_2 = \frac{6\beta(2d^3\varphi^2 + 3d^2B\varphi - 1dP + B^2d - BE)}{A^2}$, $e_3 = \frac{-6\beta\{d^4\varphi^2 + 2d^3B\varphi - 2d^2P + (Bd - E)^2\}}{A^2}$. where $\eta = x - vt$.

### 3.2. *The (2 + 1)-Dimensional extended shallow water wave equation*

The shallow water equations are derived from Navier-Stokes equations and are represented by a family of hyperbolic partial differential equations which explain the flow below a pressure surface in a fluid. The shallow water equations in unidirectional form are also called Saint-Venant equations, after Adhemar Jean Claude Barre de Saint-Venant. The applications of the shallow water equations are huge in the area of ocean modeling and coriolis forces in atmosphere. In the recent times shallow water wave equation is also introduced to examine the characteristic of moist-convection in atmospheric dynamics. Let us consider the (2 + 1)-dimensional extended shallow water wave equation is of the following form:

$$u_{yt} + u_{xxxy} - 3u_{xx}u_y - 3u_xu_{xy} + \alpha u_{xy} = 0 \tag{19}$$

It is noted that the vertical velocity term does not appear in shallow water equations though this velocity does not need to be zero. This is an important difference because as for instance, the vertical velocity cannot be zero when the floor depth changes, and thus if it is zero only for flat floors. Thus, once a solution has been found, the vertical velocity can be recovered via the equation of continuity. We reduce equation (19) into the following ordinary differential equation by choosing the transformation $u(x, t) = u(\eta)$ where $\eta = x + y - vt$. Then, we obtain

$$(\alpha - v)u'' + u^{iv} - 6u'u'' = 0 \tag{20}$$

Integrating equation (20) with respect to $\eta$, we attain

$$(\alpha - v)u' + u''' - 3(u')^2 + c_1 = 0 \tag{21}$$

where $c_1$ is an integrating constant which to be determined. Now taking the homogeneous balance between the highest order nonlinear term $(u')^2$ and linear term of highest derivative $u'''$ in equation (21), we obtain $M = 1$. Accordingly, we obtain the solution of equation (21) in the following form

$$u(\eta) = a_0 + a_1(d + H) + b_1(d + H)^{-1}, \tag{22}$$

Where $a_0$, $a_1$, $b_1$ and $d$ are constant which will determined later. Replacing equations (5) and (22) into equation (21), we mention that the left hand side of equation (21) is translated into the polynomial $(G'/G)^M$, $(M = 0, 1, 2, 3, \ldots)$ and $(G'/G)^{-M}$, $(M = 1, 2, 3, \ldots)$. After that, we collect each coefficient of these resulted polynomials equal to zero and we solve these algebraic equations by the algebraic computation software, like Maple, for the constants $a_0$, $a_1$, $b_1$, $v$, $c_1$, $d$ and found three sets of solutions as follows:

Set 1:

$$d = -\frac{B}{2\varphi},\ c_1 = 0,\ v = \frac{A^2\alpha + 16P + 4B^2}{A^2},\ a_0 = a_0,\ a_1 = -\frac{2\varphi}{A},\ b_1 = -\frac{B^2 + 4P}{2A\varphi}. \tag{23}$$

Set 2:

$$a_0 = a_0,\ a_1 = 0,\ b_1 = \frac{2(d^2\varphi + Bd - E)}{A},\ v = \frac{A^2\alpha + 4P + B^2}{A^2},\ d = d,\ c_1 = 0 \tag{24}$$

Set 3:

$$a_0 = a_0,\ a_1 = -\frac{2\varphi}{A},\ b_1 = 0,\ v = \frac{A^2\alpha + 4P + B^2}{A^2},\ d = d,\ c_1 = 0 \tag{25}$$

where $\varphi = A - C$, $P = \varphi E$, and $A$, $B$, $C$, $E$ are parameters.

For set 1: When $B \neq 0$, $\varphi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, substituting the values of the constants arranged in equation (23) into equation (22), as well as equation (7) and simplifying, we obtain following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ respectively

$$u_{41} = a_0 - \frac{\sqrt{Q}}{A}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right) + n_1\left\{\frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-1},$$

$$u_{42} = a_0 - \frac{\sqrt{Q}}{A}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right) + n_1\left\{\frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-1}$$

where $n_1 = -\frac{B^2+4P}{2A\varphi}$.

In the similarly fashion, substituting the values of the constants arranged in equation (23) into equation (22), as well as equations (8)–(11) and simplifying, we attain respectively following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ as follows:

$$u_{43} = a_0 - \frac{\varphi\sqrt{-Q}}{A^2}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right) + n_1\left\{\frac{\sqrt{-Q}}{2A}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-1},$$

$$u_{44} = a_0 - \frac{\varphi\sqrt{-Q}}{A^2}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right) + n_1\left\{\frac{\sqrt{-Q}}{2A}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-1},$$

$$u_{45} = a_0 - \frac{2\varphi}{A}\left(\frac{1}{\eta}\right) + n_1\left(\frac{1}{\eta}\right),$$

$$u_{46} = a_0 - \frac{2\varphi}{A}\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\} + n_1\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-1},$$

$$u_{47} = a_0 - \frac{2\varphi}{A}\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\} + n_1\left\{-\frac{B}{2\varphi} + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-1},$$

$$u_{48} = a_0 - \frac{2\varphi}{A}\left\{-\frac{B}{2\varphi} + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\} + n_1\left\{-\frac{B}{2\varphi} + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-1},$$

$$u_{49} = a_0 - \frac{2\varphi}{A}\left\{-\frac{B}{2\varphi} - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\} + n_1\left\{-\frac{B}{2\varphi} - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-1}$$

For set 2: When $B \neq 0$, $\varphi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, replacing the values of the constants arranged in equation (24) into equation (22), along with equation (7) and simplifying, we get following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ respectively.

$$u_{51} = a_0 + n_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-1},$$

$$u_{52} = a_0 + n_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}^{-1}$$

where $n_2 = -\frac{2(d^2\varphi + Bd - E)}{A}$.

In the similarly fashion, substituting the values of the constants arranged in equation (24) into equation (22), as well as equations (8)–(11) and simplifying, we attain respectively following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ as follows:

$$u_{53} = a_0 + n_2\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{-Q}}{2\varphi}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-1},$$

$$u_{54} = a_0 + n_2\left\{d + \frac{B}{2\varphi} - \frac{\sqrt{-Q}}{2\varphi}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\}^{-1},$$

$$u_{55} = a_0 + n_2\left(d + \frac{B}{2\varphi} + \frac{1}{\eta}\right)^{-1},$$

$$u_{56} = a_0 + n_2\left\{d + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-1},$$

$$u_{57} = a_0 + n_2\left\{d + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\}^{-1},$$

$$u_{58} = a_0 + n_2\left\{d + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-1},$$

$$u_{59} = a_0 + n_2\left\{d - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}^{-1}$$

For set 3: When $B \neq 0$, $\varphi = A - C$ and $Q = B^2 + 4E(A - C) > 0$, replacing the values of the constants arranged in equation (25) into (22), along with (7) and simplifying, we get following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ respectively

$$u_{61} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\coth\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\},$$

$$u_{62} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{Q}}{2\varphi}\tanh\left(\frac{\sqrt{Q}}{2\varphi}\eta\right)\right\}$$

In the similarly fashion, substituting the values of the constants arranged in equation (25) into equation (22), as well as equations (8)–(11) and simplifying, we attain respectively following traveling wave solutions for $l = 0$ but $m \neq 0$ and $m = 0$ but $l \neq 0$ as follows:

$$u_{63} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{B}{2\varphi} + \frac{\sqrt{-Q}}{2\varphi}\cot\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\},$$

$$u_{64} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{B}{2\varphi} - \frac{\sqrt{-Q}}{2\varphi}\tan\left(\frac{\sqrt{-Q}}{2\varphi}\eta\right)\right\},$$

$$u_{65} = a_0 - \frac{2\varphi}{A}\left(d + \frac{B}{2\varphi} + \frac{1}{\eta}\right),$$

$$u_{66} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{\sqrt{P}}{\varphi}\coth\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\},$$

$$u_{67} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{\sqrt{P}}{\varphi}\tanh\left(\frac{\sqrt{P}}{\varphi}\eta\right)\right\},$$

$$u_{68} = a_0 - \frac{2\varphi}{A}\left\{d + \frac{\sqrt{-P}}{\varphi}\cot\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\},$$

$$u_{69} = a_0 - \frac{2\varphi}{A}\left\{d - \frac{\sqrt{-P}}{\varphi}\tan\left(\frac{\sqrt{-P}}{\varphi}\eta\right)\right\}$$

where $\eta = x + y - vt$.

The solutions obtained above are further general and particular values of the parameters produce solutions available in the literature.

## 4. Discussion and graphical representations

Many researchers investigated the Boussinesq equation and the $(2+1)$-dimensional extended shallow water wave equation through individual methods. As for instance, Wazwaz (Naher and Abdullah, 2012; Wazwaz, 2012), employed independently two distinct approaches to derive the standard and singular soliton, Darvishi et al. (AkbarAli, 2013; Zhang et al., 2010; Darvishi et al., 2017, 2018), applied the semi-inverse variational principle and the sine-cosine method, Wazwaz (Akbar et al., 2012; Naher and Abdullah, 2013; Naher, 2015; Alam et al., 2014; Alam and Akbar, 2015; Wazwaz, 2010, 2012; Darvishi et al., 2017, 2018) exerted the Hereman's simplified method and the Cole-Hopf transformation method, Bekir and Aksoy (Naher and Abdullah, 2013; Bekir and Aksoy, 2013), used the exp-function method to attain traveling wave solutions of the above equations. But in this article, we construct further general and new exact traveling wave solutions by using the new generalized $(G'/G)$-expansion method. The obtained solutions are further explicable and simple which can explain the mechanism of the complicated nonlinear physical phenomenon.

Graph is an essential tool to introduce the problems and explain properly the solutions of the phenomena. A graph is a visual demonstration of closed-form or statistical solutions or other information, habitually designed for comparative purposes. When performing calculations we need the fundamental insight which provide by graphs. The graphical representations clarify the physical significance of the obtained solutions of the nonlinear evolution equations throughout the Boussinesq equation and the $(2+1)$-dimensional extended shallow water wave. The graphs of the obtained solutions include the bell shaped soliton, kink soliton, singular kink soliton, singular soliton, singular periodic solution and compaction. The solution $u_2(\eta)$ shows the shape of the exact bell shaped soliton with ($-2 \le x \le 2$ and $-2 \le t \le 2$ for the parameters $\beta = 1$, $d = 10$, $A = 2$, $B = 3$, $C = 1$, $E = 1$, $\nu = -1$). The solution $u_2(\eta)$ shows the shape of the exact kink soliton with ($-5 \le x \le 5$ and $-5 \le t \le 5$ for the parameters $\beta = 1$, $d = 1$, $A = 2$, $B = 3$, $C = 1$, $E = 1$, $\nu = -1$). The solution $u_1(\eta)$ shows the shape of the exact single soliton within the interval $-15 \le x \le 15$ and $-15 \le t \le 15$ for the parameter $\beta = 100$, $A = 2$, $B = 1$, $C = 1$, $E = 1$, $\nu = -1$. The solutions $u_1(\eta)$ and $u_3(\eta)$ are singular kink soliton and with ($-10 \le x \le 10$ and $-10 \le t \le 10$ for the parameters $\beta = 10$, $A = 2$, $B = 1$, $C = 1$, $E = 3$, $\nu = -1$) and ($12 \le x \le 12$ and $-12 \le t \le 12$ for the parameters $\beta = 1$, $d = 5$, $A = 2$, $B = 2$, $C = 1$, $E = 1$, $\nu = -1$) respectively. The solution $u_{17}(\eta)$ is a shape of single soliton with ($-10 \le x \le 10$ and $-10 \le t \le 10$ for the parameter $\beta = 10$, $A = 2$, $B = 2$, $C = 1$, $E = 1$, $\nu = 1$). The solutions $u_3(\eta)$ and $u_4(\eta)$ shows the shape of the singular periodic soliton with ($-5 \le x \le 5$ and $-5 \le t \le 5$ for the parameters $\beta = 10$, $d = 10$, $A = 2$, $B = 2$, $C = 1$, $E = -1$, $\nu = -1$) and ($-10 \le x \le 10$ and $-10 \le t \le 10$ for the parameters $a_0 = 1$, $A = 1$, $B = 1$, $C = 2$, $E = 1$, $\nu = -1$) respectively. The solution $u_1(\eta)$ is the shape of compecton with ($-50 \le x \le 50$ and $-50 \le t \le 50$ for the parameters $\beta = 20$, $A = 2$, $B = 2$, $C = 1$, $E = -1$, $\nu = 1$). The solution $u_3(\eta)$ is the shape of the singular kink soliton with ($-50 \le x \le 50$ and $-50 \le t \le 50$ for the parameters $\beta = 1$, $d = 2$, $A = 2$, $B = 2$, $C = 3$, $E = 1$, $\nu = -1$). The graphical representation of the solutions which we generated is provided below:Fig. 1–7.

## 5. Conclusion

Through the new generalized $(G'/G)$-expansion method in this article, we have examined the Boussinesq equation and the (2 + 1)-dimensional extended shallow water wave equation to extract some new and further general exact traveling wave solutions and solitary wave solutions of these equations. The solutions are attained in terms of hyperbolic, trigonometric and rational functions. The general and new solutions might be useful to analyze the propagation of small amplitude long waves on the surface of shallow water, the vertical structure of the horizontal and vertical flow velocity, in computer models for the simulation of water waves in shallow seas and harbors, models including

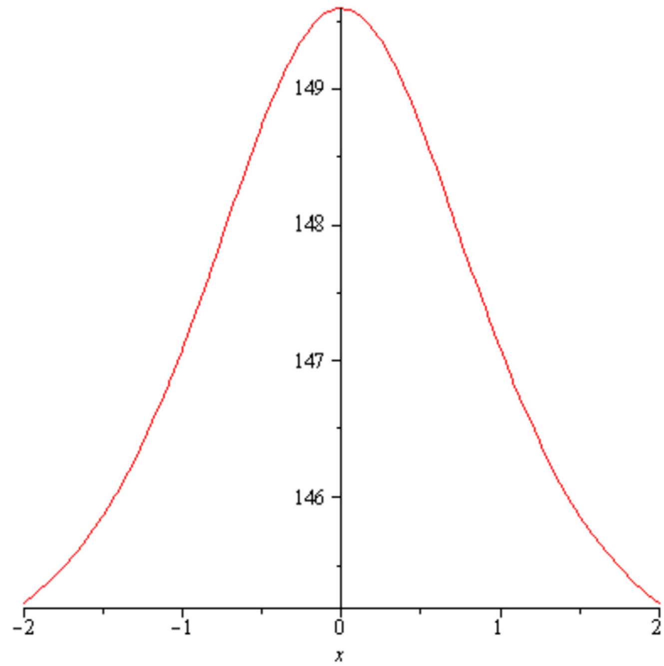


**Fig. 2.** Plot of bell shape soliton of solution $u_{22}$ within the interval $-2 \le x \le 2$ for the parameter $\beta = 1$, $d = 10$, $A = 2$, $B = 3$, $C = 1$, $E = 1$, $\nu = -1$ and $t = 0$.

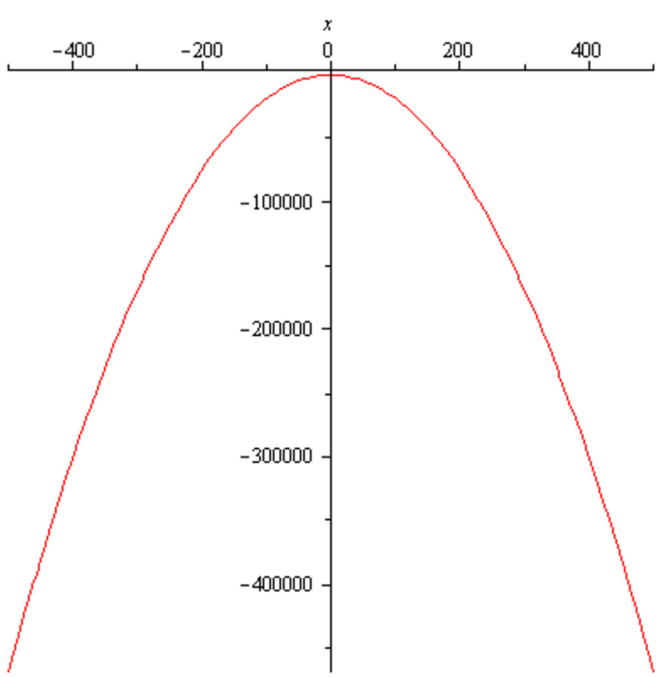


**Fig. 1.** Plot of Compacton depicted from solution $u_{15}$ within the interval $-500 \le x \le 500$ for the parameter $\beta = 20$, $A = 2$, $B = 2$, $C = 1$, $E = -1$, $\nu = 1$ and $t = 0$.

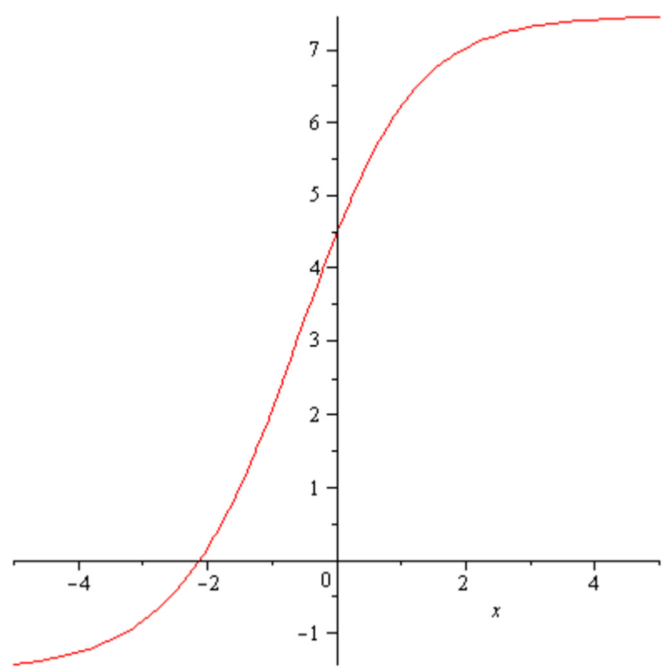


**Fig. 3.** Plot of kink shape soliton of $u_{27}$ within the interval $-5 \le x \le 5$ for the parameter $\beta = 1$, $d = 1$, $A = 2$, $B = 3$, $C = 1$, $E = 1$, $\nu = -1$ and $t = 0$.

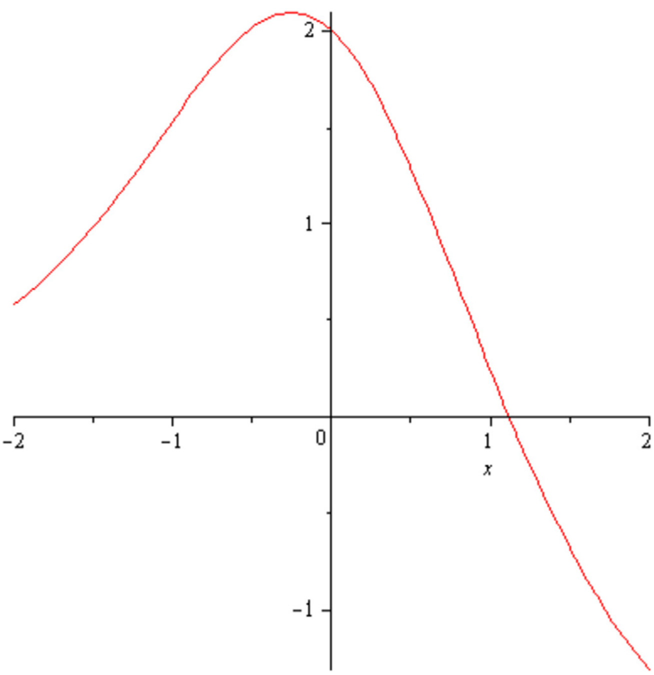


**Fig. 4.** Plot of asymmetric bell shape soliton of $u_{31}$ within the interval $-2 \le x \le 2$ for the parameter $\beta = 1,\ d = -1,\ A = 2,\ B = 2,\ C = 1,\ E = 1,\ v = -1$ and $t = 0$.

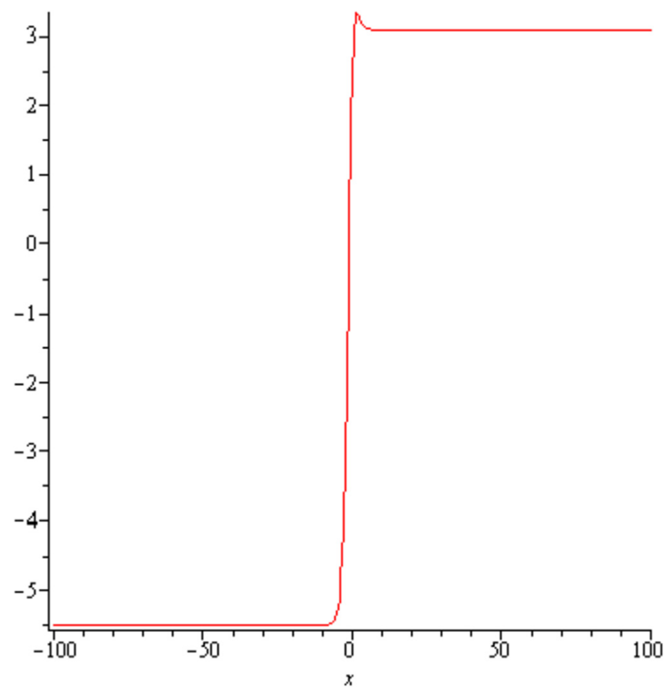


**Fig. 6.** Plot of sharp kink soliton of $u_{37}$ within the interval $-100 \le x \le 100$ for the parameter $\beta = 1,\ d = 5,\ A = 2,\ B = 2,\ C = 1,\ E = 1,\ v = -1$ and $t = 0$.

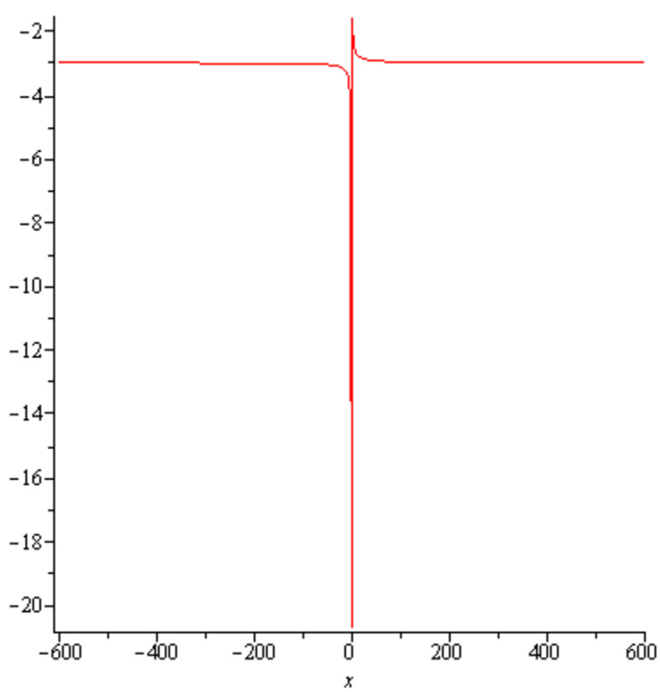


**Fig. 5.** Plot of singular kink soliton of $u_{35}$ within the interval $-600 \le x \le 600$ for the parameter $\beta = 1,\ d = 2,\ A = 2,\ B = 2,\ C = 3,\ E = 1,\ v = -1$ and $t = 0$.

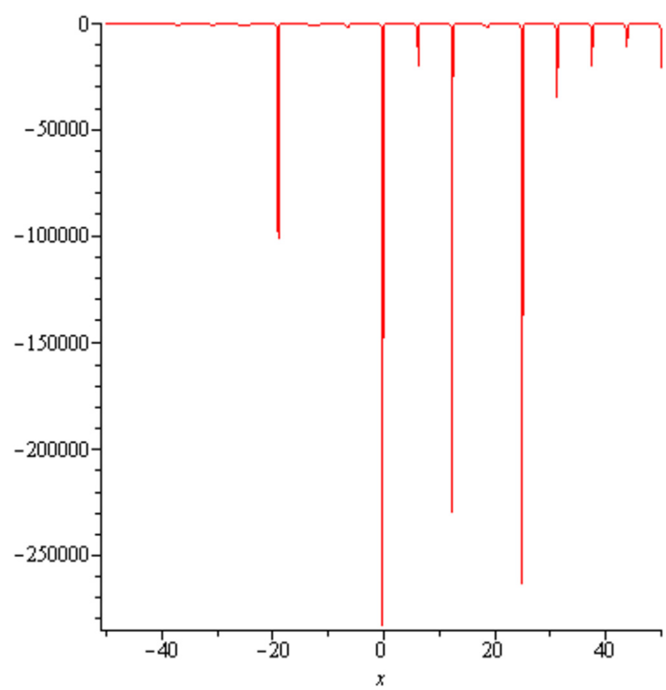


**Fig. 7.** Plot of periodic soliton of $u_{38}$ within the interval $-5 \le x \le 5$ for the parameter $\beta = 10,\ d = 10,\ A = 2,\ B = 2,\ C = 1,\ E = -1,\ v = -1$ and $t = 0$.

coriolis forces in atmosphere, the characteristic of moist-convection in atmospheric dynamics. It is noteworthy to observe that the generalized $(G'/G)$-expansion method changes the given difficult problems into simple problems which can be solved easily. The method can be implemented to other kind of nonlinear evolution equations in shallow water waves and coastal engineering to established further general and new solutions.

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