
A Linear Programming Integer Model for Cellular Manufacturing Layout Design with Machine Flexibility and Dynamic Criteria

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Abstract

The Cellular Manufacturing is adopted in batch type manufacturing industries nowadays for their production with increased productivity, less cost and time with effective control. The proposed optimization model is used to determine the cost of machine cells, i.e., machine duplication, part subcontract, inter intra cellular movements cost and cost of production associated with machine cell, such as machine reconfiguration and part inventory considering machine flexibility for various time periods. Initially, mathematical model is proposed to calculate machine cell cost with and without considering machine flexibility and then another lpp integer model is proposed to calculate the machine cell production and associated cost for the changes in time period, part type and volume considering machine flexibility. The manufacturing data in the incidence matrix and machine cell, part family data in the block diagonal form are given as input to the optimization programming language Cplex and the output are given for the two mathematical models. The data related to machine duplication, part subcontract, inter intra cellular movement; machine reconfiguration and part inventory are given. Two dimensional shop floor layouts are presented in rectilinear coordinates for all the problems for easy analysis of material movement length and shop floor area

Keywords: Cell layout design, Exceptional elements, Machine duplication, Machine flexibility, Part type and demand, Part subcontract

Introduction

In a Cellular Manufacturing System (CMS), identical components are grouped as families and related machines are formed as cells so that one part family can be manufactured within a machine cell. The CMS aims at increasing productivity and production efficiency by reducing throughput times. Machine-cell formation is to bring dissimilar machines together and dedicate them to the manufacture of one or more part families in manufacturing industries. The Cell Formation Problem (CFP) is to identify the machine groups and part families by rearranging the initial incidence matrix into a block diagonal form, with a minimum or no number of parts travelling between cells. CMS is based on the principle that similar things should be manufactured similarly. In the context of preferred strategy 'similarity', it includes design attributes such as size, shape, etc., and/or manufacturing attributes such as length, diameter, surface finish, tolerance, etc., CMS is the taking advantage that many attributes are similar, and by grouping similar attributes, one solution can be found to a set of problems and in end, using less time and effort and instead of job sequences, tooling and machine setups being based on single-component, production planning is done for an entire group of similar components or operations.

In cellular layout, components enter the cell as raw material and go out as finished products. But this condition is achieved very rarely in real-life manufacturing situations, i.e., it is very rare to find completely independent cells. The types of components, which require

to be processed in more than one cell, are known as exceptional components or bottleneck components. The operations carried outside the cell are known as exceptional elements. A machine associated with the processing of many exceptional components is known as a bottleneck machine. The movement of material within the cell is known as intracellular movement and the movement of material between the cells is known as an intercellular movement. The presence of an exceptional component increases material handling makes the job of scheduling more difficult, effective control over the cell not possible, etc.

If machine flexibility is taken into consideration, maximum of exceptional elements can be possibly reduced. One or more operations are performed by a single machine; this machine has flexibility which can process the exceptional component in the cell where it is assigned.

If dynamic conditions such as part type and volume changes are considered over the period, the machine cells and part families are also changing with the effect of attributes of manufacturing environment. Dynamic Cellular Manufacturing System (DCMS) is used to reconfigure the manufacturing cells including part families and machine groups at successive periods. Reconfiguration is removing the existing machines from machine cells, called machine relocation, adding new machines to cells including machine duplication. Dynamic cellular manufacturing is followed in manufacturing industries where product type and demand are changing, and the resources in the cell layout can be easily changed with respect to time periods.

It is usually found necessary to maintain inventories of raw materials, work-in process goods and finished goods. A manufacturing industry do inventory because the resources are more costly or less profitable. Materials Requirement Planning (MRP) attempts to maintain the alignment between the order due date and the order need date. Production Planning (PP) means the usage of manufacturing facilities in various successive time periods with the aim of optimizing the production costs.

Importance of Optimization Model in CMS

In this approach, the main goal is to form manufacturing groups in which, some machines are located in dedicated cells associated with some similar parts based on a machine-part incidence matrix. In each cell, some processes are done on the parts by machines, so that the main goal is to increase the intra-cell operations and to decrease the number of inter-cell movements (exceptional elements). Therefore, it takes a minimum time to obtain an optimal solution for medium-sized problems while it is computationally intractable for large-sized problems.

The proposed optimization models are given with the constraints for finding machine multi capacity to carry out the operations of exceptional element to mainly reduce machine duplication. Another proposed model is used to find machine reconfiguration and part inventory over the given time periods and to cope with the changes in part variety and volume.

The cell layout will have work cells inside which machine tools are arranged in series or cross lines as per process plans. But the U-shaped layout can be preferred in the cell design which will have simultaneous in-line or cross movement of materials.

Objectives

- To reform machine cells and part families with and without considering machine flexibility,
- To optimize costs of exceptional elements and production such as machine duplication, parts subcontract, inter cellular movement, intra cellular movement, machine reconfiguration and parts inventory considering the machine flexibility in dynamic manufacturing conditions such as changes in time, part demands and varieties,
- To plot facility cell layout considering material movement length, floor area.

In this paper, in section 2, literature review about various costs optimization models related to manufacturing cell operating cost optimization and facility cell layout design is given compared to the models of the recent and last decades. In section 3, a linear programming mathematical model is proposed to reduce the cost exceptional elements with and without considering machine flexibility. In section 4, a novel cost optimization model is proposed for finding the cost of production in manufacturing cell for successive time periods including exceptional elements, reconfiguration and inventory over cost reduction literatures given in recent times. In section 5, the sizes of cells, positioning cells, as well as machines, are determined with the help of the sort of cells and sort of machines concerning the origin and finally, two dimensional shop floor layouts are designed for all the benchmark problems.

Literature survey

Review on cell formation costs optimization models

This section of the review is aimed to evaluate the elimination of costs directly and indirectly related to exceptional elements.

A bi-objective possibilistic nonlinear mixed-integer programming model was presented in uncertain situations to have a suitable CMS with the aim of minimizing the total costs and total inaction of workers and machines, simultaneously. In this context, the demand for each product with a specific quality level and linguistic parameters such as product quality level, worker's skill level, and job hardness level on machines were considered with fuzzy logics, (Hashemoghli *et al.* 2019). A two-stage stochastic programming approach was followed to consider the uncertainty and to generate the problem. The objective function was to minimize the summation of production, subcontracting, material handling, and machine idleness costs. The model was considering simultaneous multiple routings and subcontracting. (Mahootchia *et al.* 2018). The objective function was minimizing total expected cost consisting of machinery depreciation cost, operating costs, inter-cell material handling cost, intra-cell material handling cost, machine relocation costs, setup costs, and production planning cost. This model determined optimum cell formation and optimum lot size (Khannan *et al.* 2016). The proposed model was decreasing the material handling cost and also increasing the machine used in a cell (Tamal Ghosh *et al.* 2014). A nonlinear programming model was proposed in significant dynamic conditions which reduce the cost of the estimated demands for inter/intracellular movements of elements (forward and backward movements), the presence of exceptional elements, intercellular dislocation of machines, and cellular reconfiguration and operational costs and initial cost of the machinery (Amir *et al.* 2018). The fuzzy multi-objective parametric programming is used to minimize the cost of exceptional element elimination, to minimize the number of outer cell operations, and to maximize utilized machine capacity (Arikan & Gungor 2005). This paper processed the problem of designing cellular manufacturing systems incorporating several design features including multi-period production planning, sequence of operations, alternate process routings, intra-cell layout, dynamic system reconfiguration,

duplicate machines, machine capacity, lot splitting, and material flow between machines in a dynamic environment (Mohammad Mahdi Paydar et al., 2013). An integrated mathematical model of the multi-period cell formation in a dynamic cellular manufacturing system (DCMS) is proposed with the aim of getting the optimal cost for it (Narendra Mohan., Srinivasa Rao, 2014) The above review on recent methodologies yielded that cost elimination of EE must look into intra cell movements and intercell movements significantly by correlating machine duplication and part subcontract respectively and for changes in part type and volume, it have to consider the machine reconfiguration and part inventory over different planning horizon and machine flexibility for eliminating exceptional machines .

Review on cell layout design

A competent cell layout was prepared which is essential to minimize the total inter-cell part travels. This research was intended to focus on an adapted mathematical model of the layout design problem considering material handling cost and closeness ratings of manufacturing cells. An efficient NP-hard technique based on a Simulated Annealing metaheuristic was proposed (Ghosh & Dan 2012). The sustainable configurations of DCMS were focused by proposing the following mathematical models, chosen appropriate social and environmental criteria, integrated them into mathematical models, and studied the impact of these criteria (Farzad Niakan 2015).

A new integrated approach was presented to solve the facility layout problem with unequal areas in an open field. Initially, the arrangement of departments adjacent to each other was determined by an improved zone algorithm; in the end, the layout obtained was improved by using a linear programming model, and the final locations of the departments were thus determined (Mohamadi *et al.* 2019). The optimum layout was designed considering different layouts like with the objective of minimizing the cost by reducing the total travelling distance of materials processed in the industry through ARENA simulation software to produce better layout design for the industry and its ability (Anbumalar *et al.* 2014). A layout optimization model is formulated based on fuzzy demand and machine flexibility and then developed a genetic algorithm (Xiaodong *et al.* 2018). It is commonly known that facility layout design is used to determine the arrangement, location, and distribution of machines in a manufacturing facility to achieve minimization of make-span time, maximization of productivity (Tsehaye *et al.* 2018). The outcome from the review showed that the facility cell layout must be an effective space saver and in reducing the material movement lengths and must take into consideration the dimensions of workstations with the aisle space and also flexible in nature for easy machine multi period reconfiguration.

Mathematical model formulation considering machine flexibility

Assumptions for optimization model and layout design

The proposed model is considering multi-period production planning, machine flexibility, inter cell movement, intra-cell movement and cell reconfiguration along with alternative processes routings.

1. The manufacturing system is considering production in a number of time periods. One time period could be a week, a month, a season, or a year.
2. The manufacturing time of operations for alternative process plans are acquired as input data.
3. One or more job sequences of all operations for all part types over machines are known.
4. The current, future demands, and batch sizes of parts are known for fixed periodic intervals, and no. of batches are also calculated.

5. The purchase price and the duplication budget are known for all machine types.
6. Subcontract price; inter-cell and intra cell movement costs are known for all parts by considering that parts are moved between the cells and in the cells, in batches. Parts are subcontracted as a whole equal to demand to carry out one or more operations and not as a finished part.
7. The entire demand of each part has to be completed in the manufacturing within the period and the quantity of production is pre-deterministic with respect to demand, processing time and production volume.
8. The machines to be duplicated, parts to be subcontracted, inter and intra movements are taken as decision variables.
9. Some machines can process one or more operations (i.e., machine flexibility). Likewise, each operation can be done by one or more machine types at different times.
10. Parts are moved between and within cells. Inter-cell movement is incurred whenever consecutive operations of the same part type are carried out in different cells. The intra-cell movement is incurred whenever consecutive operations of the same part type are processed in the same cell.

Notations used in mathematical model

Indices

- i = 1..., M (machines index),
- j = 1..., C (components index),
- k = 1..., c (cells index)
- p = 1..., P (process plans index)
- t = 1..., T (time periods index)

Parameters

- A Aisle between machines or machine to side walls,
- A_i Cost of machine type i ,
- a_{ij} 1 if part j assigned to machine i , 0 otherwise,
- B_{ik} Budget allowed duplicating the bottleneck machine i ,
- BM, BP Set of pairs of bottleneck machines, bottleneck parts (i, j)
- BS_{1x} and BS_{1y} No. of future batches of part x and y rounded to a whole, D_{1x}/V_x , D_{1y}/V_y
- BS_x and BS_y No. of batches of part x and y rounded to a whole, D_x/V_x , D_y/V_y
- C_i Periodic capacity of machine type i ,
- C_k Number of jobs in the k th cell,
- D_j Periodic demand for part j ,
- DN_i, SC_j Set of pairs of duplicated machines, subcontract parts (i, j)
- D_x, D_y Demand of part type x and y per period
- EE No. of exceptional elements;
- R_{ci} Cost of installing and uninstalling one machine type i
- h_j Inventory holding cost per unit part type i during each period.
- IA_j Intracell moving cost for a unit of part j within one cell,
- I_j Intercell moving cost for a unit of part j between two cells,
- M_k Number of machines in the k th cell,
- MT_{ij} Machining time of machine i required for part j ,
- PBS_j Ratio of no. of batches of part j , BS_j / BS_{1j}
- PQ_{jt} Quantity produced of part j in time period t
- Q_i Number of machine type i required to process parts in machine cells (integer),
- R_{ik} Number of machine type i to be purchased for cell k (integer),

S_j	Subcontracting cost of a part j for a process,
Sp_{xj}, Sp_{yj}	Process sequences of part j in machines x and y respectively;
UC_{ij}	Usage capacity of machine i for part j ($MT_{ij} D_j / C_i$),
V_x and V_y	Batch size of part type x and y per period
X_{ik}, Y_{jk}	1 if machine i and part j occurs in cell k respectively, 0 otherwise

Mathematical linear programming cost optimization model

The objective of the proposed linear programming model is the reduction of the EE costs. The inferences from literature review are the costs of EE because of inter intra movements; duplication and subcontract are most of recent crisis in the manufacturing sector. If only duplication is the remedy for an exceptional element, then there will be no intercellular movements, hence it is the need to include intracellular movements in objective function as the proposal in this work over the cost’s elimination given by (Arikan & Gungor 2005). The proposal in this model is the inclusion of the budget constraint to set the limit for the machine duplication. The other proposal is considering multi-capacity of machines. If a machine can perform more than one operation, this machine multi-capacity or machine flexibility is used to reduce the exceptional element, particularly used to eliminate the machine duplication of bottleneck machines.

Decision variables

Z_{ijk} - Number of intercell movements required by part j when machine i not available in cell k,

W_{ijk} - Number of intra-cell movements required by part j w.r.t to machine i in cells(s) k,

O_{ijk} - Number of units of part j to be subcontracted when machine i not available in cell k,

M_{ijk} - No. of machine i dedicated by duplication to cell k for producing exceptional part j.

Step 1: The objective function is to maximize the sum of the savings by either on duplicating the exceptional machines or subcontracting the exceptional parts in the original cell.

The objective function is to maximize the savings by,

Minimizing

$$\sum_k^c \sum_i^M \sum_j^C (A_i \cdot M_{ijk}) + \sum_k^c \sum_i^M \sum_j^C (I_j \cdot Z_{ijk} \cdot B_{sj}) + \sum_k^c \sum_i^M \sum_j^C (S_j \cdot O_{ijk} \cdot D_j) + \sum_k^c \sum_i^M \sum_j^C (I A_i \cdot W_{ijk} \cdot B_{sj}) \tag{1}$$

Equation (1) is an objective function that is to minimize machine duplication cost, intercellular movement cost, parts subcontracting cost, and intracellular movement cost.

Step 2: The **constraints** for bottleneck machines, machine flexibility, intra-cell movements, and bottleneck parts concerning subcontract as well as intercell movements originally assigned to the same cell are,

$$X_{ik} - Y_{jk} + U_{ijk} - V_{ijk} = 0 \quad \forall i, j, k \tag{2}$$

$$\sum_k^c \sum_i^M \sum_j^C MT_{ij} x D_j \leq C_i \quad \forall i, j, k \tag{3}$$

$$\sum_i^M \sum_j^C M_{ijk} \leq R_{ik} \quad \forall i, j, k \tag{4}$$

$$\sum_i^M \sum_j^C C_i / (MT_{ij} X D_j) \geq Q_i \quad \forall i, j \tag{5}$$

$$\sum_k^c \sum_i^M \sum_j^C M_{ijk} x MT_{ij} x D_j \leq C_i \quad \forall i, j, k \tag{6}$$

$$\sum_k^c \sum_i^M \sum_j^c \text{if}(X_{ik} = 1 \ \&\& \ Y_{jk} = 0 \ \&\& \ Inc = 1) \quad \forall i, j, k \quad (7)$$

$$U_{ijk} = 1, V_{ijk} = W_{ijk} = 0;$$

$$\sum_k^c \sum_i^M \sum_j^c \text{if}(X_{ik} = 0 \ \&\& \ Y_{jk} = 1 \ \&\& \ Inc = 1) \quad \forall i, j, k \quad (8)$$

$$V_{ijk} = W_{ijk} = 1, U_{ijk} = 0;$$

$$\sum_k^c \sum_i^M \sum_j^c \text{if}(X_{ik} = 1 \ \&\& \ Y_{jk} = 1) \quad \forall i, j, k \quad (9)$$

$$W_{ijk} = 1;$$

$$\sum_k^c \sum_{i,a}^M \sum_{j,b}^c \text{if}(M_{Fabk} == Seq_{ijk}) \quad \forall i, j, k, a, b \quad (10)$$

$$U_{ijk} = 0, W_{abk} = 1;$$

$$\sum_k^c \sum_i^M \sum_j^c \text{if}(U_{ijk} = 1 \ \&\& \ X_{jk} = 1 \ \&\& \ Bik \leq (D_{jk} * S_{jk} * 52 * 5)) \quad (11)$$

$$M_{ijk} = 1, O_{ijk} = 0, Z_{ijk} = 0;$$

$$\sum_k^c \sum_i^M \sum_j^c \text{if}(V_{ijk} = 1 \ \&\& \ Y_{jk} = 1 \ \&\& \ Bik > (D_{jk} * S_{jk} * 52 * 5)) \quad (12)$$

$$O_{ijk} = 1, Z_{ijk} = 1, M_{ijk} = 1 ;$$

$$\sum_k^c \sum_i^M \sum_j^c M_{ijk} \times A_i \leq B_{ik} \quad \forall i, j, k \quad (13)$$

$$X_{ik}, Y_{jk}, U_{ijk}, V_{ijk}, IN_{ijk}, DN_{ijk}, SC_{ijk} = 0 \text{ or } 1 \quad (14)$$

$$R_{ik}, Q_i = \text{integer} \quad (15)$$

Equation (2) is ensuring each machine and component is assigned in one cell only. Equation (3) ensures that the sum of machining times of operations in each machine is within the capacity. Equation (4) is to check that machines to be duplicated in each cell to process the part are less than the total number of duplicated machines of the same type in the cell. Equation (5) is to ensure a number of each machine type is within its utilization capacity otherwise its number will increase. Equation (6) is to ensure that the sum of machining times of operations in duplicated machines of various parts in a cell is less than its capacity. Equation (7) and Equation (8) are stating conditions to assign values for U_{ijk} and V_{ijk} as 0 or 1 as well as W_{ijk} as 0. Equation (9) is the condition to assign W_{ijk} as 1. Equation (10) is for ensuring any machine flexibility (multi-capacity) for same job sequence of bottleneck machine and to assign the value to IN_{ijk} and U_{ijk} . Equations 11, 12 are assigning 0 or 1 to M_{ijk} , Z_{ijk} and O_{ijk} . Equation 13 is setting upper limit, budget for duplicating machine in each cell.

Input Data

The incidence matrix of size $M \times N$ is the primary data input given as $Inc[i][j]$ (Refer Table 5). Manufacturing Time $MT[i][j]$, Machine Flexibility $MF[i][j]$, Job sequence $Seq[i][j]$ are also given. Block diagonal form is considered for input as machine cell $X[i][k]$ and part family $Y[j][k]$ in terms of 0 and 1 (Refer figure 1) in such a way that chosen machine/part is falling in a particular machine cell/part family, it is taken as 1, 0 otherwise. The purchase price, machine duplication budget, the capacity of each machine type are given as $A[i]$, $B[i]$, $C[i]$. Intercell moving cost, intra cell moving cost, subcontract price, part, present and future demands, and production volume of each part type are given as $I[j]$, $IA[j]$, $S[j]$, $D[j]$, $D1[j]$ and $V[j]$. (Refer Table 3.23).

Step 3: If an exceptional part is assigned to two or more exceptional machines, then either all of these machines or none are duplicated in the cell to which the part was originally assigned.

Step 4: The constraint for duplication budget is formulated using procure cost associated for those machines related to each bottleneck part.

Numerical Illustration 1

This illustration 1 is worked out for the present time period t2 for the selected three benchmark problems of small and moderate in size given in machine component incidence matrices.

By using the cell formation PARI algorithm (Ramesh et al, 2021), a similarity coefficient matrix heuristic approach, the machine cells and part families are formed and given in block diagonal form. The part incidence matrix with manufacturing data, costs and machine flexibility and the block diagonal form are used as input data for the optimization model. For the first three problems, the input data are referred from the num. illustration 2.

Benchmark problem 1: 5 Machine X 7 Component

(Input source Hachicha et al, 2007)

The incidence matrix as in table 2 is transformed using similarity coefficient heuristic algorithm, machine cells are formed after 3 iterations and part families are formed after 2 iterations. The final block diagonal form is obtained as in figure 1

Benchmark problem 2: 6 machines - 8 components

(Input source Albadawi et al, 2005)

Table 3 can be referred for incidence matrix. It is understood that there is no change in machine cells and part families and the number of exceptional elements before and after the approach are 3. Machine cell 1: 1, 3, 6; Machine cell 2: 2, 4, 5, Part family 1: 1, 2, 5, 6, 8; Part family 2: 3, 4, 7; Bottleneck machine is 2 and Bottleneck part is 6.

Benchmark problem 3: 7 machines – 11 components

(Input source Hachicha et al, 2007)

Table 4 can be referred for incidence matrix. Finally, number of exceptional element is identified as 1.

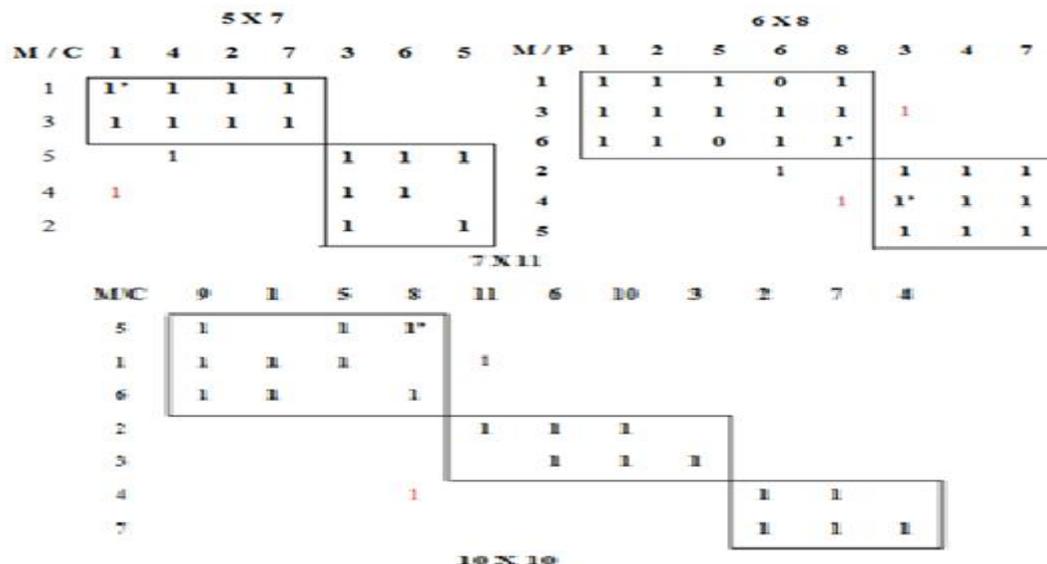


Figure 1 Block diagonal forms - Current time period t=2

The multi capacity of machines in each cell are utilised to considerably reduce the exceptional elements particularly from duplicating the bottleneck machines. The exceptional operation is coinciding with any void element of machine multi capacity in the job sequence of the incidence matrix; this is eliminating the corresponding exceptional element.

Table 1 Computational results: LPP OPL Ilog Cplex 12.8 model solutions

Benchmark Problems	Considering machine multi-capacity	min Z ₹	Z _{ijk} (Z Nos.)	M _{ijk} (M Nos.)	O _{ijk} (O Nos.)	W _{ijk} (W Nos.)	EE (DM, SC)
Benchmark Problem1 5M X 7P	Without machine flexibility	14,806	Z ₄₁₁ =9 Z ₅₄₁ = 3	M _{ijk} =0	O ₄₁₁ =3300 O ₅₄₁ =1100	∑W=14 (W ₁₁₁ = 15; W ₁₂₁ = 7, W ₃₁₁ =9; W ₂₃₂ =6 ; W ₄₃₂ =6)	2 (DM-0, SC-2)
	With machine flexibility	4,906	Z ₅₄₁ = 3	M _{ijk} =0	O ₅₄₁ =1100	∑W=15 (W ₄₁₁ =9, , W ₁₁₁ = 15; W ₁₂₁ = 7, W ₃₁₁ =9; W ₂₃₂ =6 ; W ₄₃₂ =6)	1 (DM-0, SC-1)
Benchmark Problem 2 6M X 8P	Without machine flexibility	4,765	Z ₂₆₁ =5 Z ₃₃₂ =5 Z ₄₈₁ =5	M _{ijk} =0	O ₂₆₁ =5 O ₃₃₂ =5 O ₄₈₁ =5	∑W=20 (W ₁₁₁ = 5; W ₂₂₃ = 5, W ₅₃₂ =5)	3 (DM-0, SC-3)
	With machine flexibility	1,215	Z ₂₆₁ =5	M _{ijk} =0	O ₂₆₁ =5	∑W=22 (W ₁₁₁ = 5; W ₂₂₃ = 5, W ₅₃₂ =5, W ₃₃₂ =5, W ₄₈₁ =5)	1 (DM-0, SC-3)
Benchmark Problem 3 7MX11P	Without machine flexibility	78,030	Z ₁₄₈ =5	M ₁₁₁₂ =1	O ₁₄₈ =360	∑ W =19 (W ₅₉₁ = 5; W ₃₆₂ = 3, W ₄₂₃ =6)	2 (DM-1, SC-1)
	With machine flexibility	2,635	Z ₁₄₈ =5	M _{ijk} =0	O ₁₄₈ =360	∑ W =20 (W ₁₁₁₂ =5; W ₅₉₁ = 5; W ₃₆₂ = 3, W ₄₂₃ =6)	1 (DM-0, SC-1)

Discussion of results

The multi-capacity of machines is considered to eliminate the exceptional elements. This machine flexibility is significantly helps in reducing the unwanted duplication of machines. If any void elements and machine flexibility are found in the machine cells, that particular machine flexibility operation eliminates the respective exceptional machine from duplication. The machine flexibility is used in reducing most of the costs of exceptional elements such as, machine duplication, part subcontract and intercellular movement. From the above table, considerable cost of exceptional elements is reduced compared to cell formation without machine flexibility.

In the analysis of exceptional elements, sometimes subcontracting the bottleneck parts will be dealt only because to check whether the bottleneck machines are to be considered for duplication or not.

Mathematical model in dynamic conditions

In the analysis of cost of cell formation, apart from exceptional elements, reconfiguration and inventory are taking vital role in various time periods with changes in part demand and variety. Machine Reconfiguration and parts inventory are considered important in CMS because the manufacturing is carried out along with the time phase for changes in parts varieties and volume. In dynamic conditions, during various successive time periods, machines are relocated as reconfiguration and changes in part demand should be managed as part inventory.

Inventory control is a planned approach of determining what to order, when to order and how much to order and how much to stock so that costs associated with buying and storing are optimal without interrupting production and sales. Part inventory ensure adequate supply of products to customer and avoid shortages as far as possible and make sure that the financial investment in inventories is minimum (*i.e.*, to see that the working capital is blocked to the minimum possible extent).

Inventory uses Material requirement planning (MRP) for a system of planning and scheduling the time phased material requirements to release materials and receive materials that enable the production schedule to be executed. Thus, the master production schedule is the driving force for material requirements planning. MRP provides information such as due dates for components that are subsequently used for shop floor control. Once this information is available, it enables managers to estimate the detailed requirements for each work centres.

Cell production cost optimization model

In addition to EE costs decision variables mentioned earlier, decision variables for machines adding and removing to and from cells and part inventory are,

$N_{ik}^+(t)$ = No. of machine type i added to cell k in period t

$N_{ik}^-(t)$ = No. of machine type i removed from cell k in period t

$Iv_j(t)$ = Inventory quantity of part type j kept in the period t and carried to period $t+1$

Objective function is to maximize the savings by,

Minimizing

$$\begin{aligned} & \sum_t^T \sum_k^c \sum_i^M \sum_j^c (A_i \cdot M_{ijk}) + \sum_t^T \sum_k^c \sum_i^M \sum_j^c (I_j Z_{ijk} B S_j) \\ & + \sum_t^T \sum_k^c \sum_i^M \sum_j^c (S_j O_{ijk} D_j) + \sum_t^T \sum_k^c \sum_i^M \sum_j^c (I A_i W_{ijk} B S_j) \\ & + \frac{1}{2} \sum_t^T \sum_k^c \sum_i^M \sum_j^c [RC_i (N_{ik}^+(t) + N_{ik}^-(t))] \\ & + \sum_t^T \sum_k^c \sum_i^M \sum_j^c (h_j Iv_j(t)) \end{aligned} \tag{16}$$

Equation (16) is an objective function which is used to minimize machine duplication cost, intercellular movement cost, parts subcontracting cost, intracellular movement cost, cell reconfiguration cost and inventory cost.

In addition to equations (2) to (15), the **constraints** for machine cell reconfiguration and part inventory w.r.t. time periods are,

$$N_{ik}(t-1) + N_{ik}^+(t) - N_{ik}^-(t) = N_{ik}(t), \quad \forall i, k, t \tag{17}$$

$$Iv_{jt} = Iv_{j(t-1)} + PQ_{jt} - D_{jt} \quad \forall j, t \tag{18}$$

$$Iv_{jt}^+ \leq Iv_{jt}, \quad Iv_{jt}^- \geq -Iv_{jt}, \quad Iv_{jt} = 0 \quad \forall j, t \tag{19}$$

Equation (17) is called as an equilibrium constraint ensuring that the number of machines in the current period is equal to the number of machines in the previous period, plus the number of machines being installed, and minus the number of machines being uninstalled.

In other words, it ensures the feasibility of machines over the horizon and plays the role of the memory for available machine types during the planning horizon.

Equation (18) indicates the equilibrium inventory constraint between periods for each part type at each period. It means that the inventory level of each part at the end of each period is equal to the inventory level of the part at the end of the previous period plus the quantity of production minus the part demand rate in the current period. Equation (19) determines the inventory and backorder level of each part type at each period. Obviously, the total demand of all part types over the horizon planning must be satisfied during the horizon planning.

The machine flexibility is given as job sequence similar to the part incidences mentioned in single quote. Reconfiguration cost which is almost equal in both installation and uninstallation of machines is given for each machine type. Inventory cost which includes inventory, carrying and ordering is given for each component as input.

Numerical illustration 2

The problems are solved to find out the costs of duplication, subcontract, and intra, inter cellular movements, reconfiguration and inventory with considering the machine flexibility for the past time period t1 and current time period t2for first three bench mark problems.

Benchmark problem 1: 5 Machine X 7 Component

Table 2: 5machine X 7component part incidence matrix with production data

M/C	1		2		3		4		5		6		7		A _i	B _i	Rci	C _i
	t1	t2																
1	1(3)	0	2(4)	3(3)	0	3(1)	1(2)	0	0	2(1)	0	2(3)	1(3)	0	60000	120000	4000	5760
2	0	2(3)	0	1(4)	2(4)	0	0	1(4)	1(3)	0	0	0	0	1(2)	50000	100000	3000	5760
3	2(5)	3(4)	1(4)	0	0	2(3)	3(5)	0	0	3(2)	0	1(2)	2(5)	0	120000	240000	7000	5760
4	1(3)	0	0	2(2)	1(5)		0	2(1)	0	1(3)	1(4)	0	0	2(1)	300000	600000	160000	5760
5	0	0	0	0	3(4)	0	2(6)	0	2(6)	0	2(6)	0	0	3(2)	240000	480000	120000	5760
li	4.5		3.5		4.5		4		5		3.5		4					
IAi	5		4.5		5		4.5		5		4		4.5					
Si	3.75		3.5		3.25		4		4.25		3.75		3.5					
h _j	7		8		6		7		7		7		8					
D _j	3000	3300	4200	3000	3600	2000	2400	1100	3000	1800	2000	1700	3000	4600				
V _j	500	400	600	420	400	320	400	400	300	420	400	340	300	500				

The machine flexibility is considered to reduce the exceptional elements.

Benchmark problem 2: 6 machines - 8 components

Table 3 6machines – 8 components part incidence matrix

M/C	1		2		3		4		5		6		7		8		A _i	B _i	R _{ci}	C _i
	t1	t2																		
1	0	2(1)	1(2)	3(1)	0	0	2(4)	0	0	2(1)	0	0	0	0	2(4)	3(1)	80000	160000	5000	5400
2	3(1)	0	0	0	1(3)	1(3)	0	3(3)	3(2)	0	1(1)	3(1)	1(2)	3(1)	0	0	75000	150000	4000	5350
3	1(2)	1(3)	3(2)	2(1)	3(2)	2(1)	0	0	1(2)	1(4)	2(3)	1(4)	0	0	0	2(1)	60000	120000	3500	5200
4	0	0	0	0	4(2)	0	3(2)	2(1)	0	0	0	0	2(4)	1(4)	1(1)	1(4)	120000	240000	8000	5400
5	0	0	2(4)	0	0	3(4)	1(1)	1(3)	0	0	0	0	0	2(1)	3(1)	0	80000	160000	3000	5200
6	2(4)	3(4)	0	1(4)	2(4)	0	0	0	2(3)	0	3(1)	2(1)	0	0	0	0	80000	160000	3000	5400
I _j	4	3	4	3	5	2	3	3	3	2	3	3	3	2						
IA _i	4	3	3	2	3	3	3	3	2											
SI	3	2	5	4	3	5	3	2												
h _j	8	8	8	9	9	9	8	7												
D _j	400	360	460	420	400	350	420	400	520	550	3800	360	400	420	450	440				
V _j	80	80	60	90	80	70	70	80	100	110	60	80	60	90	70	90				

Benchmark problem 3: 7 machines – 11 components

Table 4 7 machines – 11 components Part incidence matrix

M/C	1		2		3		4		5		6		7		8		9		10		11		A _i	B _i	R _{ci}	C _i
	t1	t2																								
1	0	1(3)	2(4)	0	3(1)	0	0	0	2(1)	2(1)	0	0	0	0	2(1)	0	2(1)	2(1)	0	0	0	2(1)	75000	150000	3000	5200
2	0	0	1(2)	0	1(2)	0	0	0	1(5)	0	0	2(1)	0	0	3(1)	0	3(1)	0	0	2(1)	0	1(3)	60000	120000	2000	5400
3	2(3)	0	0	0	0	1(4)	1(3)	0	0	0	3(1)	1(3)	0	0	0	0	0	0	1(3)	1(3)	0	0	120000	240000	8000	5400
4	1(2)	0	0	2(1)	0	0	3(1)	0	0	0	1(4)	0	0	1(3)	0	1(4)	0	0	0	0	0	0	80000	160000	3200	5400
5	0	0	0	0	0	0	2(4)	0	0	1(3)	0	0	1(3)	0	0	1(4)	0	1(3)	2(5)	0	1(5)	0	80000	160000	3300	5200
6	0	2(4)	0	0	2(5)	0	0	0	0	0	0	0	2(1)	0	0	2(1)	0	3(1)	3(2)	0	2(4)	0	50000	100000	1800	5400
7	0	0	3(1)	1(3)	0	0	0	1(3)	3(1)	0	0	0	0	2(1)	1(5)	0	1(2)	0	0	0	0	0	40000	80000	1600	5400
I _j	4	3	5	7	2	3	4	3	4	3	4	2	5													
IA _j	3	6	4	5	2	3	2	4	5	3	2															
S _j	6	5	8	7	6	7	9	6	5	8	6															
h _j	8	7	8	8	7	9	7	7	8	7	8															
D _j	330	300	210	240	350	350	210	360	360	320	250	280	350	400	250	240	300	250	420	320	360	300				
V _j	60	50	70	70	60	50	70	50	70	50	70	50	70	50	70	60										

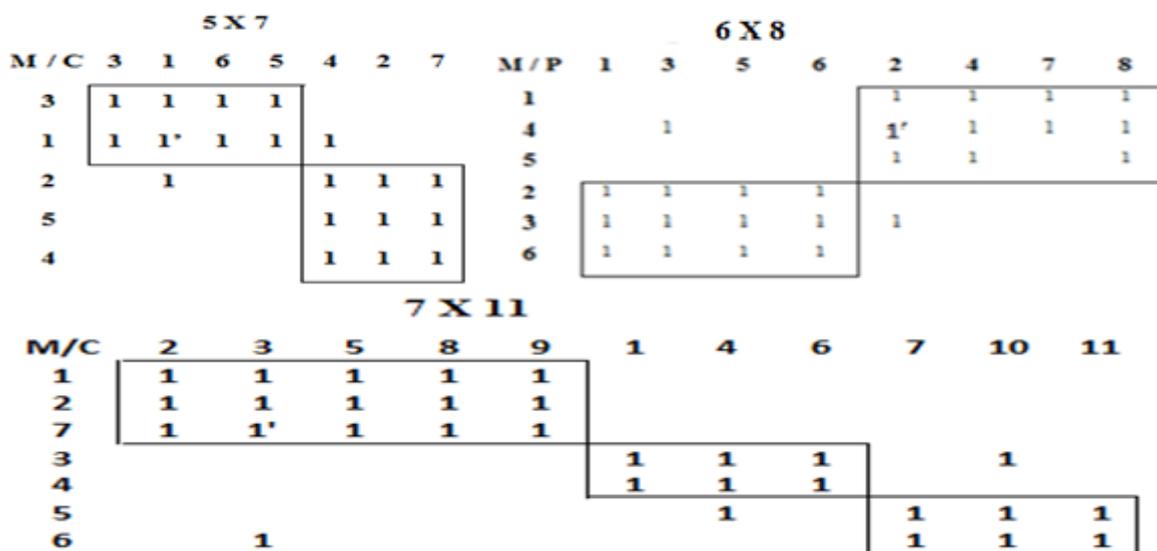


Figure 2 Block diagonal forms for past time period t=1

**Optimization model CPLEX OPL output for past and current time periods t1, t2
 Production plan with inventory for periods 1 and 2.**

Table 5 Bench mark problem 5machinesX7components:

Quantity wrt period	Time period t1						
	P1	P2	P3	P4	P5	P6	P7
PQ _{jt1}	2500	3200	4000	2400	3300	2400	3000
I _{jt-1}	800	1000	0	200	0	0	100
I _{jt1}	300	0	400	200	300	400	100
D _{jt1}	3000	4200	3600	2400	3000	2000	3000
Quantity wrt period	Time period t2						
	P1	P2	P3	P4	P5	P6	P7
PQ _{jt2}	3600	3360	2240	1200	1680	1700	4500
I _{jt1}	300	0	400	200	300	400	100
I _{jt2}	0	360	640	300	180	400	0
D _{jt2}	3300	3000	2000	1100	1800	1700	4600

Table 6 Bench mark problem 6machinesX8components:

Quantity wrt period	Time Period t1							
	P1	P2	P3	P4	P5	P6	P7	P8
PQ _{jt1}	320	480	400	420	500	420	360	490
I _{jt-1}	100	0	20	40	30	0	50	0
I _{jt1}	20	20	20	40	10	-40	10	40
D _{jt1}	400	460	400	420	520	380	400	450
Quantity wrt period	Time Period t2							
	P1	P2	P3	P4	P5	P6	P7	P8
PQ _{jt2}	320	450	350	320	550	400	450	450
I _{jt1}	20	20	20	40	10	-40	10	40
I _{jt2}	20	50	20	40	10	0	40	50
D _{jt2}	360	420	350	400	550	360	420	440

Table 7 Bench mark problem 7machinesX11components:

Quantity wrt period	Time Period t1										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
PQ _{jt1}	360	200	420	350	360	300	350	250	250	350	300
I _{jt-1}	0	50	0	20	0	0	40	0	0	0	0
I _{jt1}	60	10	70	-10	40	20	-10	10	0	30	0
D _{jt1}	300	240	350	360	320	280	400	240	250	320	300

Quantity wrt period	Time Period t2										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
PQ _j t ₂	300	200	350	280	300	250	420	250	300	420	360
I _j t ₁	60	10	70	-10	40	20	-10	10	0	30	0
I _j t ₂	30	0	0	60	-20	20	60	10	0	30	0
D _j t ₂	330	210	420	210	360	250	350	250	300	420	360

Table 8 Optimal Solution for DCMS for Problem 1 5machinesX7components

Total Cost	Machine Duplication Cost $A_i \cdot M_{ijk}$	Parts Subcontract cost $S_j O_{ijk} D_j$	Intercell Movement cost $I_j Z_{ijk} B_sj$	Intracell Movement cost $IA_i W_{ijk} B_sj$
₹39606	0	₹14000	₹36	₹1090
Reconfigure Machines removed $N_{ik}^-(t)$	Reconfigure Machines added $N_{ik}^+(t)$	Machine Reconfig. cost $RC_i (N_{ik}^+(t) + N_{ik}^-(t))$	Part Inventory $Iv_j(t)$	Part Inventory cost $h_j Iv_j(t)$
0	0	0	3580Nos.	₹24480

Table 9 Optimal Solution for DCMS for Problem 2 6machinesX8components

Total Cost	Machine Duplication Cost $A_i \cdot M_{ijk}$	Parts Subcontract cost $S_j O_{ijk} D_j$	Intercell Movement cost $I_j Z_{ijk} B_sj$	Intracell Movement cost $IA_i W_{ijk} B_sj$
₹15118	0	₹3650	₹40	₹900
Reconfigure Machines removed $N_{ik}^-(t)$	Reconfigure Machines added $N_{ik}^+(t)$	Machine Reconfig. cost $RC_i (N_{ik}^+(t) + N_{ik}^-(t))$	Part Inventory $Iv_j(t)$	Part Inventory cost $h_j Iv_j(t)$
Cell 1 – 1 Cell 2 - 2	Cell 1- 2 Cell 2 – 1	₹7750	350Nos.	₹2770

Table 10 Optimal Solution for DCMS for Problem 3 7machinesX11components

Total Cost	Machine Duplication Cost $A_i \cdot M_{ijk}$	Parts Subcontract cost $S_j O_{ijk} D_j$	Intercell Movement cost $I_j Z_{ijk} B_sj$	Intracell Movement cost $IA_i W_{ijk} B_sj$
₹99045	₹80000	₹2640	₹50	₹1175
Reconfigure Machines removed $N_{ik}^-(t)$	Reconfigure Machines added $N_{ik}^+(t)$	Machine Reconfig. cost $RC_i (N_{ik}^+(t) + N_{ik}^-(t))$	Part Inventory $Iv_j(t)$	Part Inventory cost $h_j Iv_j(t)$
Cell1 – 2, 7 Cell 2 – 4 Cell 3 – 5, 6	Cell 1 – 5, 6 Cell 2 – 2 Cell 3 – 4, 7	₹11900	410Nos.	₹3280

Discussion of results

For the problem 1, the cell layouts are same, because of changes part demand and type, part subcontract and inventory are varying and also the costs of cell formation, production associated cost are different for each time periods and no machine reconfiguration. In problem 2, reconfiguration is required because of slight modification in cells. The part demand and type are changing in time periods, it directly affect machine cell cost and associated production cost. Part subcontract, reconfiguration and part inventory costs are considerable in the total cost. In problem 3, the each cell requires reconfiguration such as machine installation and uninstallation and also part inventory is required in both the time periods. Machine duplication consumes much of total cost apart from another part subcontract. In all the cases, total part inventory are given in the table, part subcontract are given for the entire demand and movement cost are given for number of batches.

Shop floor facility cell layout

The strategies followed in facility cell design are flexibility, optimum space utilization, and minimum capital investment. Part volume to be processed takes a vital role in locating the machinery within the cell and locating the cell within the shop floor. The aisle can be considered around 0.9m to 1.5m for small and medium-size layouts and 1.5m to 1.8m for larger size layout according to the availability of floor area. But for effective material handling and supervision, the minimum lengths of the aisle 0.9m and 1.5m are preferred. In all these benchmark problems, machine width is considered as 1.2m for all machines in smaller and medium-size cellular layouts.

The machines are arranged in a U-shaped layout to have effective intra movements of materials, tools, labour, and supervision over the entire cell. The machines are divided into three sets equally and the first two full sets of machines are arranged along the right side, lengthwise and the remaining machines are arranged along the left side of the cell starting from the entry. The shop floor layout is prepared to locate cells concerning the storeroom and stock room. Raw materials are transferred from the storeroom to cells and finished parts are stocked in the stock room. The material handling systems used nowadays in cell layout are AGV, forklifts, trolleys, pallets, and bins, and most of the time by manual to and from the storeroom; the stock room and intra inter cells. Intercellular and intra cell movements are measured from the storeroom to the stock room through cells in machine clusters.

Two dimensional rectilinear layouts for past time period t=1:

The shop floor layout for 5machines X 7components problem is same for both the periods.

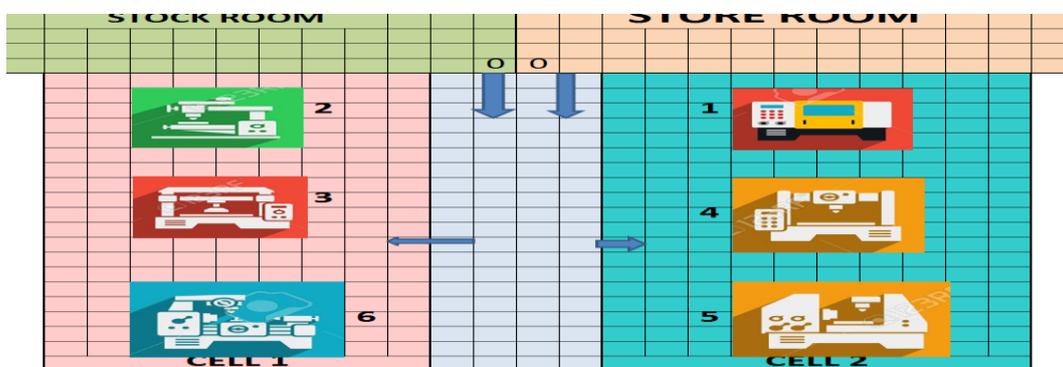


Figure 3 6machines X 8components

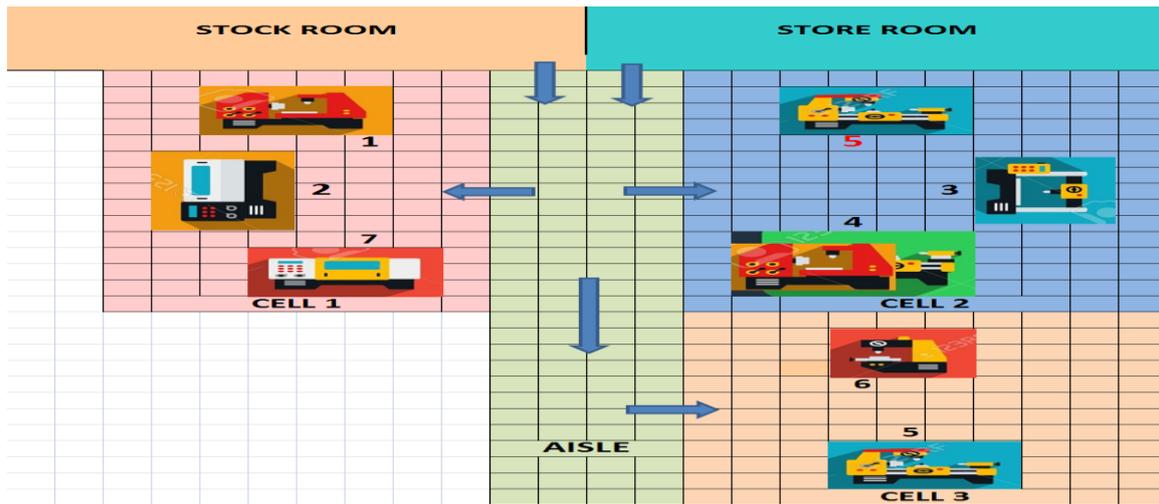


Figure 4 7 machines X 11 components

Two dimensional rectilinear layouts for current period t=2:

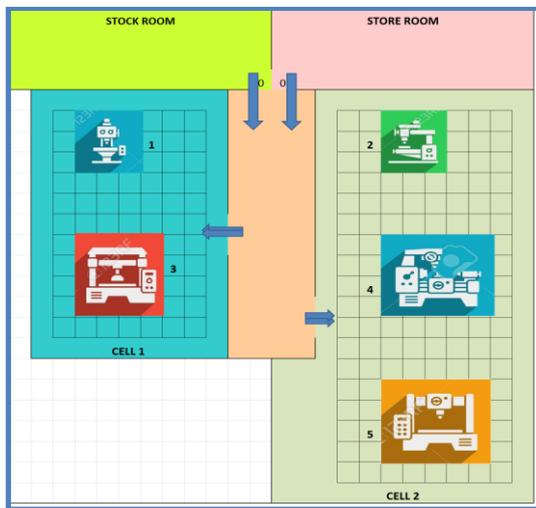


Figure 5 5 machines X 7 components

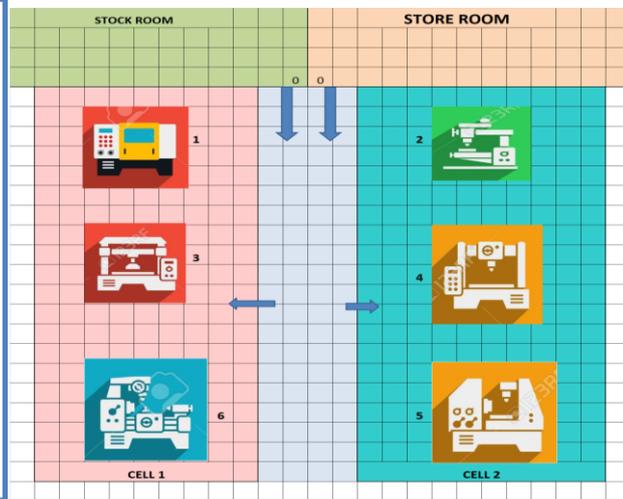


Figure 6 6 machines X 8 components

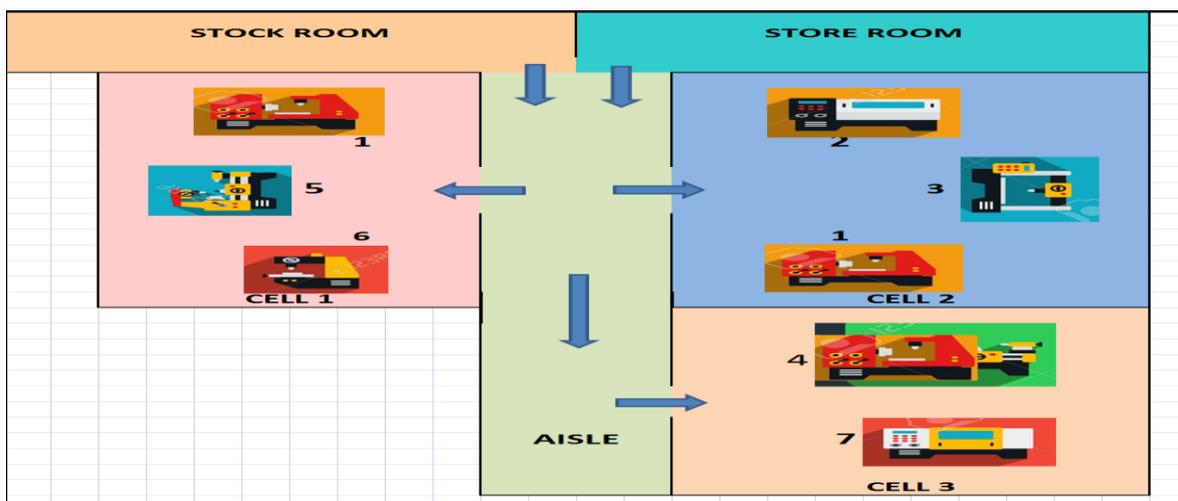


Figure 7 7 machines X 11 components

Discussion of results

The cell and machine locations are measured as rectilinear from the origin i.e., adjacent entry of stock room and store room for easy plotting of the shop floor. The aisle in between cells, machines, and the partitions are considered as per problem size. The centre passage aisle is allowed suitably related to sizes of part volume to be handled. Cell order and machine orders help locate the cells and machines in the shop floor layout. 2D shop floor layout plans are prepared with the input of cell order, machine order, and cell and machine dimensions.

Conclusion

The proposed mathematical model is used to reduce most of the exceptional elements by considering the machine flexibility. This multi capacity of machines helped in reducing the machine cell cost by avoiding the duplication of bottleneck machines. Another proposed optimization model for machine cell multi period production cost is used to reveal effectively the machine reconfiguration and part inventory data which are helpful in production plan of regular manufacturing activities in an lean cell manufacturing. The shop floor layouts are used to determine the material movement length, floor area which is helping in setting up cell layout with the capability of easy reconfiguration with respect to changes in part type and volume as well as various time periods. The optimization models can be extended for line balancing, scheduling and inbound supply chain for setting up of facility cell layouts for different cellular manufacturing.

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